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this law is itself a fact, if we construe it to mean that the principle of having something greater than itself is true of London. Thus a fact may be true of a fact. But the law, ' x is greater than y ,' while true of London, is not itself a fact. This, then, is true only in the third or existential sense. Now if it so happened that London was greater than New York, then the complex "New York is greater than London" would be annihilated altogether, unless some mind entertained it as a hypothesis. In that case it would be true only of the mind entertaining it. It would then be a *mere* hypothesis. But according to our supposition that New York is greater than London, ' x is greater than London' has a triple status; it is a fact, it is true of New York, and it is a hypothesis. When such is the case we have, I believe, the fourth type of truth, to which we shall presently proceed.

Before doing so, however, it may be worth while to indicate the sense in which nature is irrational, or non-logical. A fact which is true existentially may be a logical fact. Thus ' $(x$ is greater than y) implies (y is less than x),' while it is true of New York and London, is also itself a fact. There is such a propositional complex, quite apart from its application to these historical cities. It is doubly true; it is true existentially, as the law of something, and it is also a mathematical fact holding of the property of magnitude. A realm of existence which exemplified only logical or mathematical facts, so that all of its laws were categorically true independently of everything but the bare structural or formal properties of the propositional complex might be termed a purely rational world. It would simply illustrate pure logic and pure mathematics. As a matter of fact, however, the physical or natural laws, although they contain such formal truths, are not as they stand formally true. For, as we have seen, such a law as Galileo's law of acceleration, to the effect that the velocity of a falling body is equal to the time of its fall multiplied by a constant, is true *only* in the sense that actual falling bodies obey it.

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(To be continued)

THE LOGICAL SIGNIFICANCE OF THE PARADOXES OF ZENO

THAT the paradoxes of Zeno still maintain their interest for the student of logic is a significant fact which has been recently emphasized by the discussion of these problems in Mr. Bertrand Russell's volume, "Our Knowledge of the External World." In this work the conviction is expressed that these ancient difficulties

have now received a final solution through the formulation of a positive theory of the infinite, and this is cited as affording a brilliant illustration of the triumphs which we may expect to achieve in philosophy when the method of logical analysis comes into its own. Russell's solution turns upon the definition of a mathematical continuum as applied to space and time; the question of the nature of space and time is placed in the foreground, and the solution assumes an actual infinite. It seems to the writer, however, that the paradoxes may be regarded from a different point of view, and that, by so shifting the standpoint, the above problems are seen to be merely subsidiary and incidental. Without attempting to deny that the nature of space and time is in some sense involved (in the argument of the stadium this is the proximate, though not the ultimate, issue), it may be doubted whether any view which puts this issue in the foreground poses the problem in its most fundamental form. It seems to me at least, that an even more universal and significant issue is raised by Zeno's arguments; and that unless this is clearly grasped, the bringing in of the problem of the infinite tends rather to confuse than to clear the logical situation.

The central problem of the paradoxes is nothing less than the problem of the nature and limits of logical explanation, and consequently, of the relation between reality and logic. Russell asserts the principle that logic is confined to the formulation and invention of possibilities, as distinct from a determination and limitation of the actual.¹ But this is only one aspect of the matter, and concerns the validity of Zeno's arguments only indirectly. Russell also affirms that the analysis of change and continuity is a problem whose solution is a necessary pre-condition for understanding the world, and that this is preeminently a task for logic.² If this is true, it becomes important to apprehend clearly the nature and significance of logical explanation. For unless we do this, we may find ourselves confronted with an insoluble problem, on account of having raised an illegitimate question; we may be lured into the pursuit of an ancient will-o'-the-wisp under a modern guise. It is in their relation to this question that Zeno's arguments prove their enduring vitality; for they furnish a crucial test of the limits of logical explanation. It was no doubt Zeno's purpose to prove the unreality of space and time and motion, and of the pluralistic universe generally. It must be confessed that his arguments are not sophistical, nor capable of an easy and superficial solution. Nevertheless, they are capable of another interpretation than that which he himself put upon them; instead of implying the unreality of change, they may be

¹ "Our Knowledge of the External World," p. 58.

² *Op. cit.*, p. 16.

regarded as demonstrating the impotence of logic to construe the reality of change. This, at least, is the standpoint from which I wish to discuss them.

The argument of the race-course seeks to prove the impossibility of reaching the end of the course; it rests the proof upon the admitted necessity of first traversing the half of any remaining distance, a necessity which is never obviated. Let us expound Zeno somewhat freely, but without losing sight of the main point. "For the sake of the argument," we can imagine him saying, "I am willing to admit that the runner is able to traverse half the distance between himself and the goal; and having done this once, I will admit that he can do it again, and so on repeatedly, as long as he likes. But you on your part must also admit that he can not reach the goal until he has first made an end of traversing the successive half-distances which I have thus conceded to him; and this, you will readily see, is impossible. For there will always be another half-distance to traverse, however far along the runner may have advanced in the course of this serial progress."

What is now the root of the difficulty? It evidently consists in this, that from Zeno's concession to the requirements of the runner who wishes to reach his goal, there is no logical transition; physically, the difference may become as small as you please, but logically, the gulf is a yawning chasm. Zeno conceives and defines the runner's advance wholly within the qualitative determination: "some half distance this side of the goal"; from this there is no logical transition to the qualitatively different determination: "at the goal." The transition is a logical "leap," for it is what Aristotle calls *μεταβάσις εἰς ἄλλο γένος*. The puzzling character of the argument is due to the fact that Zeno's initial concession constitutes a quantitative approximation to the goal, which is alluring to the imagination, since the runner is constantly getting nearer and nearer, until the intervening distance seems so small that it may be neglected; but the self-constraining immanence of the initial logical assumption is then brought to bear, and we are sharply reminded that the runner's progress is defined only within the sphere of the "not-yet-there." The beauty of the argument lies in the triumphant victory of the thought over the seductions of the imagination and the blandishments of the quantitative approximation. The thought insists on respecting the boundary line between one quality and another; in spite of the fact that the confusion and weariness of mind caused by the never-ending process of diminishing the intervening distance, seeks relief in letting it be finally wiped out, the thought holds this tendency in check and immutably insists on the basic fact that Zeno has conceded its diminution by one half only.

Russell replies to this argument by pointing to the fact that there really exists something beyond an infinite series. We might just as well have replied to Parmenides's argument for the One, by pointing to the general assumption that more than one reality exists; or invited Zeno to go down to the race-course to see for himself how the runner reaches the end of the course. For the point on which Zeno is insisting is that the limit which such a series approximates is *beyond* the series, not immanent in the series, and therefore not logically implicit in it, but reached only by a leap. And it is this leap which Zeno passionately refuses to take, because for him logic is the sole standard of reality, and a reality which can not be logically construed is a mere appearance. Let us note precisely *why* the completion of the series defined in Zeno's argument is declared to be impossible. It is certainly not on the ground of empirical evidence drawn from the observation of runners. Nor is it on the ground of the fact that counting the steps of such a series one by one, or the imaginative traversal of these steps, is empirically found impossible of completion within a finite time. In the nature of the case such negative empirical evidence is unattainable. The ground is absolutely and purely logical, and does not primarily involve any reference to time at all. The ground is contained in the logical principle that a series so defined as to be endless and therefore as having no last step, can not without a manifest self-contradiction be regarded as ended, so that the last step is reached. This logical impossibility may be viewed secondarily under the form of time; it then translates itself into the proposition that the series can not be completed in a finite time. But this is because the series is in itself incomplete, and incomplete *by definition*. Zeno defines the runner's progress, so far as he is willing to concede it, in terms of a series which has the attainment of the goal outside itself; although this goal is the limit which the series endlessly approaches, the limit remains forever external to the process of approximation.

Zeno's initial assumption may indeed be regarded as sophistical, for it is no more difficult for a runner to traverse the whole than the half of an intervening distance; every actual whole is from another point of view a half. But it must be remembered that the argument is *ex concessis*; in any case, its significance does not depend on the propriety or the impropriety of the initial assumption, but in bringing to consciousness the principle that we can not construe the whole out of the half. The concession of the half proves to be illusory, since it leads to an endless approximation, and the argument very properly insists that half of the intervening distance is not the whole, no matter how small this whole may be.³

³ The argument has here been discussed from the point of view of a pro-

The essence of the argument of the race-course, and of the remaining arguments as well, can therefore be stated without any reference to the infinite. The reference to the infinite is collateral; in the first and second arguments it is introduced as the result of an apparent concession yielded by Zeno in favor of the possibility of motion. The concession turns out to be illusory, because it merely defines an operation of approximation of such a nature as always to remain within the sphere of the "not-yet-there"; the conclusion of the argument brings this to consciousness, and motion is seen to be possible only on condition that it shall never arrive anywhere. The second argument, of Achilles and the tortoise, is essentially identical with the argument of the race-course, except that the concession is here introduced in another form. Instead of the process of halving the intervening distance, we have here offered us as a concession, the process of reaching the point which the continually moving tortoise has just left. As long as the tortoise moves at all, at however slow a rate, this process is capable of repetition; and however often repeated, it does not pass over into the logically transcendent process of catching up with the tortoise.

The reflective process, specific illustrations of which are thus experimentally introduced into Zeno's two first arguments, is inherently infinite. It contains no immanent principle which logically necessitates or provides for its own cessation; it contains no immanent principle which logically necessitates or provides for a transition to a qualitatively different determination, Hegel and his dialectic method to the contrary notwithstanding. Here we have a fundamental logical truth, applying to every reflective process, however defined. This principle appears in the reflective process of subdivision as applied to spaces, times, or intensive magnitudes; it applies to the process of positing an anterior cause for an event, and the process of positing a posterior effect for a cause; it appears in the process of representing a spatial beyond, and a temporal before or after; it is the process in accordance with which the infinite series of finite numbers is produced by means of the repeated addition of *one*; it is the magic by which the mathematicians produce an infinity of ideas out of one object of thought, by means of the repeated positing of an idea of the already posited idea; it applies to the relation between a universal and its embodiment in particulars; it is therefore

gressive series, but essentially the same considerations apply if we interpret the argument as positing a regressive series, by which the runner is prevented from making a start. If, before traversing the whole of any initial distance, it is necessary that he should have first traversed the half, then, since every half is for the next reflection apprehended as a whole, he will not be able to traverse any initial distance, however small; for the argument concedes only the half of such a distance.

the secret of the inductive leap, and of the approximative character of the natural sciences, and of all the historical disciplines; it applies to the analysis of a given totality into its aspects or qualities; in short, it applies to all reflective processes however defined. The principle involved is, that if a thing can be done once, it can be done again; for the sphere of the logically possible is quite incommensurable with the sphere of the actually possible, and is not subject to the influence of the quantitative. But this absolute quantitative freedom of the reflective process is matched by an equally absolute qualitative restriction; for it can not transcend the logical immanence by which it is initially defined. The self-representative map-system which begins with England does not finally transform itself into a map of the United States, or enlarge itself to cover the universe in its scope; and the approximation-process which is defined as remaining within the sphere of the "not-yet-there" does not finally leap over into the "there." Not because there is insufficient time; this is an imperfect expression for the self-restraint of the logical immanence, and has deceived many a thinker into accepting a solution of Zeno's paradoxes based on the infinite divisibility of time, whereby any infinite process becomes commensurable with any finite time. But such a solution is in danger of missing the point entirely; it solves the problem, not by solving it, but by matching it with a parallel problem which it does not solve. The insufficiency of the time is an inadequate and misleading expression for the logical impossibility of the transition to a new quality. It is misleading, because it tends to substitute the concept of actual impossibility for the concept of logical impossibility; the supposition that a new quality could finally emerge as the immanent outcome of the reflective process is a logical absurdity, and an immanent transcendence is a contradiction in terms.⁴

But does not the positive theory of the infinite here come to our aid, and does it not throw an entirely new light upon the problem of the paradoxes? By no means; the positive theory of the infinite does not alter the logical situation here exposed in the slightest degree; if anything, it sets it into stronger relief. The advantage claimed

⁴ There is a childish game which mimics the attempt of the understanding to bridge the chasm between one kind and another by means of a quantitative approximation. It begins by counting: one I see, two I see, and so on, until it reaches ten, when it triumphantly brings out, *Ten-nes-see*, the name of a state! produced quite naturally by the process of counting. It is the very same trick or illusion which is the secret of the supposed explanatory power of the theory of evolution. Because we can discover quantitative approximations, the emergence of a new quality seems somehow less abrupt and miraculous; after a while it becomes almost self-evident, and finally the new quality seems a logical and necessary outcome of the preceding state. Cf. Kierkegaard, "*Begrebet Angest*," p. 32, where there is a similar remark.

for the newer formulation is twofold. In the first place, it defines the concept of the infinite all at once, by a single logical act, determining the class by the defining property of its members. This is both correct and necessary, for it is the condition that we should have any definition at all. But why is it an advantage as over against the negative theory? The negative theory has precisely the same merit. A definition of the infinite as endless determines the whole endless series or collection all at once, by means of a single *negative* defining property, in precisely the same way that a definition of it as self-representative, for example, determines it all at once, by means of a single *positive* defining property.⁵ The reason why the positive theory seems to be an advance in this respect is that a confusion arises between the endlessness or incompleteness of the series, with the endlessness or incompleteness of the definition of the series. The same confusion arises in connection with the static character of the concept. It is assumed that a static concept can only define states, but not processes. But this is not so. Time as pure duration, as an endless passing by, as a constant vanishing of states which *as states* are therefore non-existent, is just as much a conception of the intellect as is the conception of an instant or of a permanent state. This first advantage, then, is illusory, and is simply a circumlocution for the logical nature of all conception, positive and negative alike. The second advantage of the positive theory is that it starts from a totality posited by definition, which is by definition (directly or by implication) resolvable into an infinity of members. And this is no doubt correct and permissible; perhaps this method of approach is the key to a revolutionary advance in mathematical research, on which question I do not pretend to offer a judgment. But before I can bring myself to admit that it also revolutionizes logic and philosophy, I must ask to have one little question answered; *how* does the positive theory posit the infinite totality with which it starts? Does it construe it out of its single elements, or does it posit it by taking a new point of departure? Happily, the answer is not hard to find, for the positive theory itself not only does not conceal, but openly professes, that no infinite totality is ever reached by a summing up of elements,

⁵ Russell seeks to show by examples that infinite series are not necessarily endless, but that they may have one or even two ends. This involves a confusion as to just what it is that is in each case endless; a finite line has two ends, but the process of determining points on the line, or the process of subdividing the line (the synthesis and the analysis) is nevertheless endless. We have here a confusion of the logical problem of completing an endless process with the problem of the physical existence, and hence the finitude, *from a different point of view*, of totalities whose analysis gives rise to the endless process; in other words, it is answering Zeno by appealing to the fact that the runner can be observed actually reaching his goal. Cf. "Our Knowledge of the External World," p. 179.

but only by shifting to an absolutely new point of view. The leap which in the negative theory is outside the definition is in the positive theory included in the definition—but it is *precisely the same leap*. The positing of infinite totalities which from another point of view are finite, is an old achievement of thought; such totalities we have always with us. If there is anything remarkable or revolutionary connected with the whole matter (which is what one would be led to suppose from the romantic glamor with which the invention of the positive formulation has been invested), it would have to consist in the abolition of the *leap*; in the overcoming, by the technique of mathematics, of the qualitative immanence of logical thought. This would indeed be a remarkable achievement; Hegel staked his reputation as a thinker on the claim of having accomplished it by means of the dialectic method, but history has not judged him leniently in this respect. Take any infinite totality, a line, a duration, or the series of finite numbers; if the reflective process is applied to such a totality by way of analysis, the analysis never becomes complete, never reaches the last elements, without making a leap; if the reflective process is applied to the elements by way of synthesis, it does not reach the totality (or the continuity) except by making the same leap in the reverse direction. The endless subdivision of spaces yields spaces, but not points; the endless multiplication of points yields points, but not a geometrical continuum. The analysis or the synthesis becomes *complete* by a breach of logical continuity; points and instants are not homogeneous with spaces and times, and the one concept is not derivable from the other by a mere quantitative determination.

The positive and negative theories of the infinite thus supplement one another. They express the obverse and converse sides of the same logical situation. The positive theory leaves the inherent negativity of the infinite absolutely undisturbed (in the compact point-series there is no next point; the subdivision of spaces and times gives us no smallest element, and hence no points or instants; a transfinite number can not be reached by counting, etc.). The negative theory, properly understood, does not deny the existence of positive totalities which are infinitely resolvable into elements. Both theories unite in asserting that such totalities are, from the standpoint of the elements, reached by assuming a new point of departure; and that the elements are, from the standpoints of the totalities, reached in precisely the same way.

We need only touch briefly on the third and fourth arguments, since the fundamental principle has already been amply discussed. The arrow is at rest in every position which it occupies (or in every instant); how then can it be said to move? We may reply, if we

wish, that the initial proposition constitutes an irrelevant tautology; for it is not surprising that the arrow should be at rest *in every position*, position being a determination of rest. (In the same way we may reply that motion takes time, and the instant not being a determination of time, it is a tautology to say that there is no motion *in the instant*, or even in all the instants taken collectively.) Or we may reply, if we prefer, that rest is a concept which requires more than a single instant for its determination, *and also more than a mere collection of instants*, since rest consists in occupying the same position for all the instants included *in a certain period of time*. But these answers lose all their force if there is nothing in time but instants, and nothing in motion but positions. For out of the positions which the arrow occupies, you can not construe the transition from one position to another; and out of the instants which the arrow occupies in its flight, you can not construe the duration in which alone the motion can take place. Both positions and transitions, both instants and times, are required for any adequate determination of motion; Zeno assumes only one of the two mutually supplementary concepts, makes an unsuccessful effort to construe the other, and interprets his failure as evidence of the unreality of motion.⁶

The argument of the stadium is somewhat complex in statement, and I shall not attempt to elucidate it in detail. Here again the difficulty arises from supposing time and space to be constituted by instants and points, and from the attempt to define motion and its

⁶ In this note I will refer the reader to a couple of passages in Mr. Russell's book, which seem to indicate that at certain crucial points he is unaccountably blind to the issues involved. The first is on page 151, and asserts the necessity of analyzing change completely, by reference to terms which are not changes. We have already shown that this involves a leap out of the immanence of the change-concept. This shifting to a new point of view is not a necessary consequence of reflecting upon changes, but is reached as the result of an arbitrary interruption of this reflective process, by virtue of an act of the will. The second passage is on page 138, and asserts that if the mathematical theory is adequate, nothing happens when a body moves except that it is in different places at different times. The phrase "in different places at different times" conceals the possibility of an ambiguity. If it includes in its meaning the transition from one place to another the proposition is trivial, and says nothing that needs any support from mathematical theory; to be exact, the verb "comes to be" should be substituted for the verb "is." But if not, the proposition is self-contradictory; the idea of a process, as indicated by the word "happens," is equated with the idea of a state, as indicated by the word "is." The proposition is refuted by the basic principle of the Eleatic dialectic: *You can not construe becoming from being*. Either Mr. Russell has overlooked this dialectical boundary, as impassable for logic as the widest chasm; or else he has not overlooked it, but seeks to make the leap in the strength of mathematical theory. In the latter case, his proposal is a recrudescence of the Hegelian dialectic in a mathematical form.

velocity exclusively in terms of instants and points. This is equally impossible whether we assume a finite number of points or instants in a given space or time (an assumption which is certainly false), or an infinite number of such points and instants (an assumption which is undoubtedly correct). For the points or instants do not yield space or time except in conjunction with the idea of a serial order, a before and after, a between; but this idea is simply the notion of a geometrical or temporal continuum in a surreptitious form. The assertion that there is no next point (instant) is merely an expression for the fact that space (time) is not constituted by points (instants); by which we do not mean to deny that it contains them, nor that positions and motions are determinable only by means of a reference to such entities.

The lesson of the paradoxes is thus not a mere clarification of our ideas of space and time; they do not pose an esoteric and highly technical problem, soluble by the progressive refinement of our logical and mathematical methods. They teach that the fundamental principle of all logical explanation is the principle of immanence; or in other words, of ground and consequence. Logic is therefore compelled to confess its impotence when confronted with the problem of explaining change. This does not mean that change is mere appearance; such an assertion is a perversion of the voice of logic, which philosophers have allowed themselves through misunderstanding its scope and function. Every change and every transcendence is a breach of logical continuity, and hence from the point of view of logic, a paradox. The far-reaching consequences of this principle I can not here undertake to suggest; but the history of philosophy and of science is full of attempts to dash the head of the intellect against this impenetrable wall.

In conclusion, let me try to express this principle in a slightly different form. Professor Royce and Mr. Bradley have each formulated a logical principle having a bearing upon the problem we have here discussed. Although Royce seems to regard the two formulations as in some sense antagonistic, it is possible to combine them in a single formula, and this formula will then sum up the argument of the present paper. Bradley says: *Thought can not make differences*. Royce replies: *Thought begets, as the self-evident outcome of its own unitary reflective purpose, an endless process*.⁷ I assume, and the preceding argument must be the warrant for my assumption, that these two theses must be combined; they constitute, in fact, the obverse and the converse side of the same principle. I also assume that the additional considerations which Bradley and Royce, re-

⁷ Cf. Royce, "The World and the Individual," I, p. 473, where Bradley is quoted.

spectively, regard as expansions or implications of their respective principles, must either be rejected or re-interpreted. When Bradley, for example, adds that Thought can not receive differences from the outside, and ready-made, he evidently places himself at the viewpoint of Zeno. In so far as he intends to assert that immanence is a *sine qua non* of every form of logical apprehension, our argument has sought to establish the same position; but in so far as he identifies Thought in general with logical apprehension, he ignores the function of Belief as a supplementary mode of thought. For Belief is the form in which we apprehend the transcendent. On the other hand, when Royce seeks to interpret the endlessness of the reflective process as typical of the way in which a monistic principle logically unfolds itself in an infinite multiplicity of forms (thus substituting logical emanation or development for the idea of creation as an explanation of the world of space and time), it will be necessary to urge that the endlessness of the reflective process is always a repetition of the same; that it never passes over into a new quality; that its apparent "movement" is precisely analogous to the uniform velocity with which a moving body continues to move when free from the influence of every external force; that this endlessness, like the uniform motion of the moving body, is inherently static (not involving any change) and purely ideal, and thus affords by itself no description or type of any actual happening in any actual world; and, finally, we must urge that the arguments of Zeno, who was also a monist, constitute the historically valid objection against every attempt to establish a logical bridge between the One and the Many. On the basis of this understanding I propose, as a summary of the argument contained in the preceding pages, the following unification of the two principles quoted above: *The endless quantitative approximation toward a new quality which is implicit in the process of reflection remains always within the quality initially posited, thus testifying to the impossibility of a necessary logical transition from one quality to another.*^s

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REVIEWS AND ABSTRACTS OF LITERATURE

The Idealistic Reaction against Science. A. ALIOTTA. Translated by Agnes McCaskill. London: Macmillan and Company. 1914. Pp. xxii + 483.

This work is a revision of the author's "La Reazione idealistica contro la scienza," 1912, made for presentation in English dress. Besides the

^s I owe the underlying idea of this paper to a study of Kierkegaard, whose logical position I have sketched in a paper published in a recent number of the *Philosophical Review*.